THEORETICAL AND EXPERIMENTAL INVESTIGATION OF THE FUNCTION OF THE WALL FLOW DEFLECTING RING. THE DETERMINATION OF THE OPTIMUM DISTANCE BETWEEN DEFLECTING RINGS

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Based on the earlier published mathematical model the problem has been solved of the optimum distance between the wall flow deflecting rings. These rings of small width placed near the wall of a packed column provide for the equality of the integral mean densities of irrigation in region near the column wall and in the bulk of the packing. An equation has been derived for the optimum distance between the wall flow deflecting rings convenient for practical calculations. A practical example has been used to show that the wall flow deflecting rings substantially improve the distribution of liquid in the column.

The use of the wall flow deflecting rings (WFDR)[1] of small width placed near the column wall appears to be effective means of checking the extent of the wall flow.

With a suitable size of these rings one can achieve indepence of the intensity of mass transfer of the column diameter[2] and hence also safe scale up.

The investigation has been so far directed at the formulation of an adequate mathematical model of liquid flow in the presence of the WFDRs. Initially[3], we have shown the applicability of the principal equations and boundary conditions on an example of a column with a single WFDR. As the next step[4,5] we have worked out a mathematical model valid for the case of a number of WFDRs and solved it for various boundary conditions accounting for the effect of the WFDRs on the distribution of the liquid flow. It has been shown that in the most general case the liquid that hits the WFDR drains from the WFDR not only on its inner periphery, but, instead, mostly leaves *via* the packing pieces contacting the inner periphery of the WFDR. The model appears adequate provided that the boundary conditions, reflecting the above mechanism of liquid draining from the WFDR, are formulated with the aid of the probability theory starting form the equal probability of all spatial orientations of the packing pieces[5].

As a logical next step appears the determination of the optimum distance between the WFDRs. In fact, this optimum distance depends not only on the characteristics of the packing but also on the thermodynamic and kinetic parameters of the process. This, of course, gives rise to various alternatives of the optimization problems the solution of which calls for different approaches.

The aim of the present paper is to investigate the conditions of equal mean velocities of phases in the wall region and the bulk of the packing. This problem is of importance for almost all equilibrium processes – primarily the rectifications. Strong variations of the velocity over the column cross section lead to variable thermodynamic conditions in different zones of the packing and hence to considerable imparing of the separation process. Of particular importance these effects become during operation near the equilibrium, *i.e.* in the product refinement and in the rectifications near the minimum reflux.

The Mathematical Model

The principal equation of flow distribution under axial symmetry takes in the dimensionless coordinate the following form[6]:

$$\frac{\partial^2 f(r, z)}{\partial r^2} + \frac{1}{r}\frac{\partial f(r, z)}{\partial r} = \frac{\partial f(r, z)}{\partial z}. \qquad (1)$$

The boundary condition[7] near the column wall reads:

$$-\frac{\partial f(r, z)}{\partial r} = B[f(r, z) - CW)], \quad r = 1. \qquad (2)$$

The quantities B and C depend on the type of the packing and the relation between the size of the packing and the column diameter. The constants have been determined experimentally for certain types of packings[8].

The solution takes then the form:

$$f(r, z) = A_0 + \sum_n A_n \, J_0(q_n r) \exp(-q_n^2 Z), \qquad (3)$$

where

$$A_0 = C/(1 + C) \qquad (4)$$

and where q_n designates the roots of the equation:

$$\left(\frac{2C}{q_n} - \frac{q_n}{B}\right) J_1(q_n) + J_0(q_n) = 0. \qquad (5)$$

The coefficients A_n in Eq. (3) depend on the initial distribution of liquid in the zone of the given WFDR and remain constant in the segment between two neighbouring WFDRs. They are determined gradually starting from the topmost WFDR (see Fig. 1) using the following recurrent formula[5]:

$$A_n^{(k+1)} = \frac{2(q_n^2/B - 2C)^2}{[(q_n^2/B - 2C)^2 + q_n^2 + 4C]\,J_0^2(q_n)} \left\{ \frac{C r_1}{(1 + C)\,q_n} J_1(q_n r_1) + \right.$$

$$+ \sum_{\substack{m' \\ m \neq n}} r_1 A_m^{(k)} \exp(-q_m^2 z_0) \frac{1}{q_m^2 - q_n^2} [q_m J_0(q_n r_1) J_1(q_m r_1) - q_n J_0(q_m r_1) J_1(q_n r_1)] +$$

$$+ A_n^{(k)} \exp(-q_n^2 z_0) \frac{r_1^2}{2} [J_0^2(q_n r_1) + J_1^2(q_n r_1)] +$$

$$\left. + \frac{1}{2}\left[1 - \frac{C r_1^2}{1 + C} - 2 r_1 \sum_n A_n^{(k)} \exp(-q_n^2 z_0) \frac{J_1(q_n r_1)}{q_n}\right] \int_{r_1 - d}^{r_1} \varphi(r)\, J_0(q_n r)\, dr \right\}. \quad (6)$$

The process of draining of liquid from the WFDR is described by the distribution probability density of radius $\varphi(r)$ behind the integral in Eq. (6). For its determination

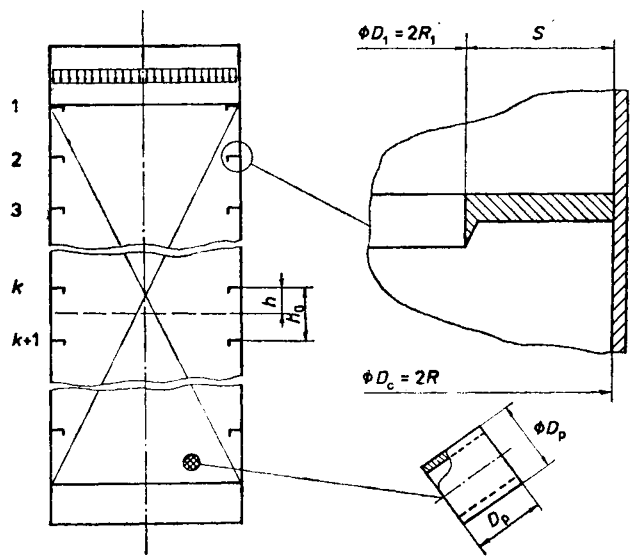

FIG. 1

Scheme of the column equipped with the wall flow deflection rings and a detail of the wall flow deflecting ring we have derived the following expressions using the probability theory[5]:

$$\varphi(r) = \frac{2}{d\pi} \ln \frac{d + \sqrt{[d^2 - (r_1 + r)^2]}}{r_1 - r} \quad \text{for} \quad r_1 - d \leqq r < 1 - d \tag{7}$$

$$\varphi(r) = \frac{2}{d\pi} \ln \frac{(1 - r)\{d + \sqrt{[d^2 - (r_1 - r)^2]}\}}{(r_1 - r)\{d + \sqrt{[d^2 - (1 - r)^2]}\}} + \\ + \left[1 - \frac{2}{\pi} \arcsin \frac{1 - r}{d}\right] \frac{S}{(1 - r)^2} \quad \text{for} \quad 1 - d \leqq r \leqq r_1 \,. \tag{8}$$

Optimization of the Distance Between the WFDRs

Since gas is usually well distributed over the column cross section the problem of ensuring uniform flow conditions in the wall region of the column usually reduces to ensuring uniform density of irrigation in the volume of the column. Mathematical modelling of the distribution of liquid with the aid of the adopted model (*1*)–(*8*) as well as the experiment show that there exists a region in the column with essentially uniform distribution of liquid. Substantial changes occur only in the proximity of the wall in region approximatelly four times the particle diameter wide. Apart from this the width of this region and the profile of the distribution of the density of irrigation vary along the length between the WFDRs (see the continuous lines in Fig. 2). From the standpoint of the aim of this paper it appears therefore useful to investigate the integral means of the density of irrigation along the height of the bed between two neighbouring WFDRs and over the column cross section in the bulk of the bed and in the wall region. The criteria of optimization may then be expressed as follows:

$$\bar{f}(z_0) = \frac{\int_0^{z_0} \left[1 - \int_0^{1-t} 2rf(r, z) \, \mathrm{d}r\right] \mathrm{d}z}{z_0[1 - (1 - t)^2]} = 1 \,, \tag{9}$$

where t designates the width of the wall region and $\bar{f}(z_0)$ the volume integral mean density of irrigation in the given zone.

A question here arises as to the width of the wall region that should be adopted. Clearly the value of this width shall affect the resulting value of optimum distance z_0.

The packing in the model is viewed as a pseudohomogeneous medium with the exception of the probabilistic modelling of the flow distribution applied in the derivation of Eqs (7) and (*8*). Here we have taken into account effects concerning a single packing element. The final result, however, has the form of continuous random variables, which formally agrees with the concept of a pseudohomogeneous medium.

However, a single element of a random packing realizes practically perfect mixing of liquid. Consequently, the bed may be looked upon as a matrix of zones of perfect mixing. This necessarily causes its discrete character. For this reason and rigorousness of mathematical treatment the size of a zone in the optimization problem must not be smaller than this discrete limit. The maximum width of the wall zone shall be equal to the nominal size of the packing element, *i.e.* $t = d$. In order to clarify the effect of the width of the wall zone on the course of optimization of the distance z_0, the optimization was carried out for $t = d$ as well as for $t = 2d$. In addition, we have determined also the accuracy of the determination of the optimum distance under the condition $f(z_0) = 1 \pm 0{\cdot}01$. In all cases it was found that a more definite value of z is obtained for $t = d$. The deviation of the optimum distance found for $t = d$ from that found for $t = 2d$ was less than the accuracy of its determination. Consequently, considering $\bar{f}(z_0) = 1 \pm 0{\cdot}01$, z_0 found for $t = d$ appears optimal also for $t = 2d$.

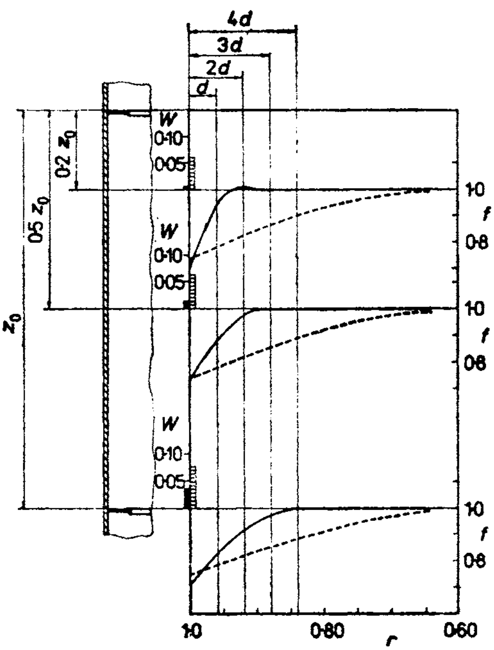

FIG. 2

Profiles of the density of irrigation and the wall flow at three levels between the wall flow deflecting rings. Continuous lines indicate the profiles in the column equipped with WFDR, broken lines in the column without WFDR. Black rectangular boxes indicate the magnitude of the wall flow in the column with WFDR, empty boxes in the column without WFDR

The expression (*9*) takes then, after analytical integration, the following form

$$\bar{J}(z_0) = \frac{1 - (C/(1 + C))(1 - d)^2}{1 - (1 - d)^2} +$$

$$+ \frac{2(1 - d)}{z_0[1 - (1 - d)^2]} \sum_n A_n^{(k)}(z_0) \frac{1}{q_n^3} J_1(q_n(1 - d)] [\exp(-q_n^2 z_0) - 1] = 1 \,. \quad (10)$$

The obtained equation was solved on a computer using the Newton method.

From Eq. (*10*) it can be seen that the optimum distance depends on the coefficients A_n, which in turn depend on the sequence number of the WFDR. For this reason we have performed a numerical test in order to assess this effect. Table I shows the results of this test as well as the relative deviation between each pair of adjacent WFDRs. From the table it is seen that after the third WFDR the error is small and after the fifth WFDR the error does not exceed 0·2%. However, with the increasing sequence number of the WFDR the amount of computation increases. In order to save the effort all computations were carried out for the sequence number $k = 5$.

The Expression for the Optimum Distance of the WFDRs

Considering the awkward expressions and the amount of necessary computation involved in the optimization routine it is desirable to find a simpler expression for the optimum distance suitable for practieal calculations. In order to achieve this goal have utilized the theory of dimensions.

In the most general case the optimum distance (h_0) depends on the coefficient of liquid spread (D), the size of the packing element (D_p), column diameter (D_c) as well as the boundary condition parameters B and C. Considering the correlation for the parameter C having the form[8]

$$C = KD_c/D_p \quad (11)$$

TABLE I

Optimum distance between WFDRs computed for several sequence numbers and percentual variation

k	z_0	%	k	z_0	%
1	0·3293	28·8	5	0·4339	0·14
2	0·4244	1·51	6	0·4345	0·12
3	0·4308	0·46	7	0·4350	0·07
4	0·4328	0·25	8	0·4353	—

one can write that

$$h_0 = f(D, D_c, D_p, K, B)\,. \tag{12}$$

In the model equations (1)–(8) as well as in Eq. (10) all quantities were rendered dimensionless using the column radius R. This is not very suitable for the purpose of obtaining the general expression for the optimum distance of the WFDRs. On the one hand, the effect of the WFDR is dominant in the wall region of the packing where the characteristic dimension of the process is the dimension of the packing element (D_p). Accordingly, one can expect only a weak influence of the column radius on the optimum distance of the WFDRs. On the other hand, due to certain peculiarities of the solution of the optimization problem on a computer using Eq. (12) with respect to D_p is an order of magnitude smaller than when one uses the dimensionless form based on R. Taking this into account the general equation takes the form

$$Z = \frac{h_0 D}{D_p^2} = n_1 (D_c/D_p)^{n_2} (S/D_p)^{n_3} K^{n_4} B^{n_5}\,. \tag{13}$$

A large number of calculations have been carried out covering practically the whole possible range of dimensionless variables:

D_c/D_p: 8–100 $\qquad$ K: 0·1–0·4

S/D_p: 0·2–0·9 $\qquad$ B: 5–8 .

Processing further these data using the least square method numerical values of the constants were obtained with which Eq. (13) takes the following form

$$Z = 0{\cdot}124 (D_c/D_p)^{0.229} (S/D_p)^{1.096} K^{-1.770} B^{-0.31}\,. \tag{14}$$

The mean arithmetic deviation of the equation amounts to 1·25% while the mean square deviation was 2·33%. The maximum error of the optimum distance was 5·3% which corresponds to maximum deviation $\bar{f}(z_0) = 1{\cdot}01$. This means that in all cases the maximum nonuniformity of the integral mean density of irrigation does not exceed 1%. The confidence limits of the constant at the $\alpha = 0{\cdot}05$ significance level are as follows:

$$n_1 = 0{\cdot}124 \begin{matrix} +\,0{\cdot}041 \\ -\,0{\cdot}031 \end{matrix} \qquad n_4 = -1{\cdot}770 \pm 0{\cdot}060$$

$$n_2 = 0{\cdot}225 \pm 0{\cdot}030 \qquad n_5 = -0{\cdot}310 \pm 0{\cdot}14\,.$$

$$n_3 = 1{\cdot}096 \pm 0{\cdot}053$$

As has been anticipated the effect of the relative column diameter measured in terms of the characteristic packing size is not strong. In fact the functional dependence (*11*), (ref.[8]), which is the principal source of the effect of D_c/D_p, was studied only for $D_c/D_p \leqq 25$. It may be thus expected that for large values of the column to particle diameter ratio the indicated proportionality is not valid as the value of the exponent n_2 may be lower than the found one. Considering the relatively wide confidence limits for the exponent over B and the relatively narrow range of values of this parameter (for the Raschig rings – 7·0, for spheres – 6·7; see ref.[8]) one can propose a simpler form of the correlation shown below

$$Z = 0{\cdot}068(D_c/D_p)^{0.23}\,(S/D_p)^{1.1}\,K^{-1.77}\,.$$

In this case the mean arithmetic deviation was 3·6% and the mean square deviation was 5·9%. The maximum error amounted to 10% which causes the maximum nonuniformity of the integral mean density of irrigation equal to 2·8%.

CONCLUSIONS

The use of WFDRs spaced along the column length in optimum distances one from another substantially improves the hydrodynamic conditions in the column. This has been shown in Fig. 2 for a concrete example: $z_0 = 0{\cdot}434\,.\,10^{-2}$, $D_c/D_p = 50$, $S/D_p = 0{\cdot}8$ and the packing of Raschig rings. The continuous lines show the profile of the density of irrigation on three levels corresponding to $0{\cdot}2z_0$, $0{\cdot}5z_0$, and z_0. The black boxes in this figure show the magnitude of the wall flow. It may be seen that the variations of the profile of the density of irrigation take place within the zone about $4d$ wide. In this case $\bar{f}(z_0) = 0{\cdot}99966$ or, in another words, the nonuniformity of the flow is less than 0·1%. For comparison the figure shows by broken lines and empty boxes the density of irrigation profile and the wall flow in the same column without WFDRs. In this latter case $f = 0{\cdot}4$, which means that the volume integral mean density of irrigation in the wall region is by 60% less than in the bulk of the packing.

This indicates that the effect of the WFDRs, in spite of their narrow width, is so large that convincingly justifies their use.

Eq. (*14*) alone, independently of its simple form, very well agrees with the earlier published results[9]. In this example for the parameters $D_c = 200$ mm; $D_p = 25$ mm; $S = 5$ mm and the packing of Raschig rings ($K = 0{\cdot}181$, $B = 7{\cdot}0$, $D = 2{\cdot}25$ mm) the optimum ditance obtained was 106 mm which compares very favourably with the experimentally foud optimum value 100 mm.

LIST OF SYMBOLS

A_0, A_n	coefficients in Eq. (3)
B, C	dimensionless parameters of the boundary condition (2)
D	coefficient of radial spread of liquid, m
D_c	column diameter, m
D_p	diameter of packing piece, m
$d = D_p/R$	dimensionless diameter of packing piece
$f = L/L_0$	dimensionless density of irrigation
$\bar{f}$	volume integral mean density of irrigation, Eq. (9)
h	packed bed depth measured from the lower WRDR, m
h_0	distance between packing elements, m
J_0, J_1	Bessel function first kind, zero and first order
K	coefficient in Eq. (12)
k	sequence number of WFDR counted from the top
L, L_0	local, respectively mean density of irrigation, $m^3/(m^2 s)$
m, n	summation index
$n_1, \ldots, n_5$	constants in Eq. (13)
q_n	roots of Eq. (5)
R	column radius, m
R_1	internal radius of WFDR, m
r'	radius, m
$r = r'/R$	dimensionless radius
$r_1 = r'_1/R$	dimensionless internal radius of WFDR
S	width of WFDR, m
$s = S/R$	dimensionless width of WFDR
t	dimensionless (related to column radius) width of the wall zone
W	dimensionless wall flow (related to total flow rate)
$z = hD/R^2$	dimensionless coordinate of height
$z_0 = h_0 D/R^2$	dimensionless distance between WFDR
$Z = h_0 D/D_p^2$	dimensionless variable
α	significance level
$\varphi(r)$	probability density for the distribution of radius r

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